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A GENERALIZED DESCRIPTION OF THE SPHERICAL THREE-LAYER RESONATOR WITH AN ANISOTROPIC DIELECTRIC MATERIAL

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SUMMARY

The eigenvalue equation of a three layer spherical resonator with arbitrary material parameters and an anisotropic dielectric inner sphere is described. The axial-symmetric modes of the resonator, the field distributions, the Q-factors and applications are discussed.

INTRODUCTION

Spherical isotropic resonators have been discussed in the literature since nearly 80 years /1/-/10/. Primarily the famous work of Debye /1/ must be mentioned here, because most of the theoretical questions in connection with spherical isotropic resonators have been solved already in this classical work. J.Broc /3/ in 1950 intensively studied the case of the spherical resonator formed by two concentric metallic spheres, M.Gastine et al. /5/, /6/ analyzed the free dielectric sphere and Affolter et al. /7/, /8/ described the spherical dielectric resonator enclosed in a metallic sphere.

Spherical resonators for many reasons are interesting and can be used in different applications:

- The spherical cavity is the cavity with the highest possible Q-factor because its volume to surface ratio is the biggest of all possible cavity stuctures
- Spheres can be produced easily; this is especially true in the case of dielectric spheres with small diameters for applications in MICs
- The excitation of the electromagnetic fields in a spherical cavity is easily done
- The isotropic spherical resonator has a threefold degenerated eigenresonance which can be used for special applications as will be shown.

In this paper a generalized theory shall be given, which describes a spherical structure of three concentric spheres with arbitrary isotropic material parameters; additionally it will be assumed, that the inner sphere can be an uniaxially anisotropic dielectric material. The eigenvalue equation of the electromagnetic fields in this structure will be derived, the eigenfrequencies, field distributions, possible modes and the Q-factors shall be discussed. Finally the applications of spherical resonators will be discussed.

THE ELECTROMAGNETIC FIELD OF THE ANISOTROPIC SPHERE

It is assumed that the material properties of the inner uniaxially anisotropic dielectric sphere can be described by the permittivity tensor:

$$\overleftrightarrow{\varepsilon} = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \tag{1}$$

Let E' and H' be the electric and the magnetic field strength, respectively, then reduced field quantities E and H are introduced by:

$$\vec{E} = j\sqrt{\varepsilon_0}\,\vec{E}' , \qquad \vec{H} = \sqrt{\mu_0}\,\vec{H}' \tag{2}$$

to simplify the field equations. Then the electromagnetic field inside the sphere can be described by Maxwells equations in spherical coordinates:

$$\frac{\delta}{\delta\vartheta}(\sin\vartheta H_\varphi) - \frac{\delta}{\delta\varphi} H_\vartheta =$$
$$k_0 r \sin\vartheta (E_r(\sin^2\vartheta(\varepsilon_1\cos^2\varphi+\varepsilon_2\sin^2\varphi)+\varepsilon_3\cos^2\vartheta)$$
$$+ E_\vartheta(\sin\vartheta\cos\vartheta(\varepsilon_1\cos^2\varphi+\varepsilon_2\sin^2\varphi-\varepsilon_3))$$
$$+ E_\varphi(\sin\vartheta\cos\varphi\sin\varphi(\varepsilon_2-\varepsilon_1))) , \tag{3}$$

$$1/\sin\vartheta \frac{\delta}{\delta\varphi} H_r - \frac{\delta}{\delta r}(r H_\varphi) =$$
$$k_0 r \,(E_r(\sin\vartheta\cos\vartheta(\varepsilon_1\cos^2\varphi+\varepsilon_2\sin^2\varphi-\varepsilon_3))$$
$$+ E_\vartheta(\cos^2\vartheta(\varepsilon_1\cos^2\varphi+\varepsilon_2\sin^2\varphi)+\varepsilon_3\sin^2\vartheta)$$
$$+ E_\varphi(\sin\varphi\cos\varphi\cos\vartheta(\varepsilon_2-\varepsilon_1))) , \tag{4}$$

$$\frac{\delta}{\delta r}(r H_\vartheta) - \frac{\delta}{\delta\vartheta} H_r =$$
$$k_0 r \,(E_r(\sin\vartheta\sin\varphi\cos\varphi(\varepsilon_2-\varepsilon_1))$$
$$+ E_\vartheta(\sin\varphi\cos\varphi\cos\vartheta(\varepsilon_2-\varepsilon_1))$$
$$+ E_\varphi(\varepsilon_1\sin^2\varphi+\varepsilon_2\cos^2\varphi)) , \tag{5}$$

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$$\frac{\delta}{\delta\vartheta}(\sin\vartheta E_{\varphi}) - \frac{\delta}{\delta\varphi} E_{\vartheta} = k_0 r \sin\vartheta H_r \quad , \tag{6}$$

$$1/\sin\vartheta \frac{\delta}{\delta\varphi} E_r - \frac{\delta}{\delta r}(rE_{\varphi}) = k_0 r H_{\vartheta} \quad , \tag{7}$$

$$\frac{\delta}{\delta r}(rE_{\vartheta}) - \frac{\delta}{\delta\vartheta} E_r = k_0 r H_{\varphi} \quad . \tag{8}$$

If an uniaxially anisotropic material with $\varepsilon_1 = \varepsilon_2$ and if only modes, which are independent of the azimuthal angle, are considered, solutions in the form of E_{mn0}- and H_{mn0}-modes as in the case of the isotropic sphere can be found and the following coupled wave equations can be derived for the H_{φ}-components of the E_{mn0}-modes in an uniaxially anisotropic material described by the permittivities ε_1 and ε_3:

$$r\frac{\delta^2}{\delta r^2}(rH_{\varphi}) + \frac{\delta}{\delta\vartheta}\left[\frac{1}{\sin\vartheta}\frac{\delta}{\delta\vartheta}(\sin\vartheta H_{\varphi})\right] + k_0^2 r^2 \varepsilon_3 H_{\varphi} =$$

$$= k_0 r \frac{\varepsilon_1 - \varepsilon_3}{\varepsilon_1}\left[\frac{\delta}{\delta\vartheta}(f(H_{\varphi})\sin\vartheta) - \frac{\delta}{\delta r}(r f(H_{\varphi})\cos\vartheta)\right], \tag{9}$$

with $k_0^2 = \varepsilon_0 \mu_0$ and:

$$f(H_{\varphi}) = \frac{1}{k_0 r}\left[\frac{\delta}{\delta\vartheta}(\sin\vartheta H_{\varphi}) - \frac{\delta}{\delta r}(rH_{\varphi})\cos\vartheta\right] \tag{10}$$

and for the E_{φ}-components of the H_{mn0}-modes:

$$r\frac{\delta^2}{\delta r^2}(eE_{\varphi}) + \frac{\delta}{\delta\vartheta}\left[\frac{1}{\sin\vartheta}\frac{\delta}{\delta\vartheta}(\sin\vartheta E_{\varphi})\right] + k_0^2 r^2 \varepsilon_1 E_{\varphi} = 0 . \tag{11}$$

As (11) shows, the electromagnetic fields of the H_{mn0}-modes are described by the same differential equations as for the case of the isotropic sphere with a permittivity $_1$, whereas the E_{mn0}-modes are influenced by the anisotropy of the dielectric material. A solution for the H_{φ}-components described by equ. (9) can be found in form of a series expansion as given in (12):

$$H_{\varphi} = \sum_{n=1}^{\infty} A_n \Psi_n(kr) P_n^1(\cos\vartheta) \quad , \tag{12}$$

where Ψ_n are the spherical Besselfunctions and P_n^1 are the associated Legendre- polynomia and where the amplitude coefficients are not independent of each other. As further investgations show, this result can be interpreted so, that the electromagnetic field inside the anisotropic sphere can be described by a superposition of coupled $E_{m(n-2)0}$-, E_{mn0}- and $E_{m(n+2)0}$-modes of the isotropic sphere.

THE THREE LAYER SPHERICAL RESONATOR

A three-layer spherical resonator as shown in Fig.1, in which the inner spherical dielectric material may be an isotropic or uniaxially anisotropic material will be considered and the eigenvalue equation of this general spherical resonator is solved.

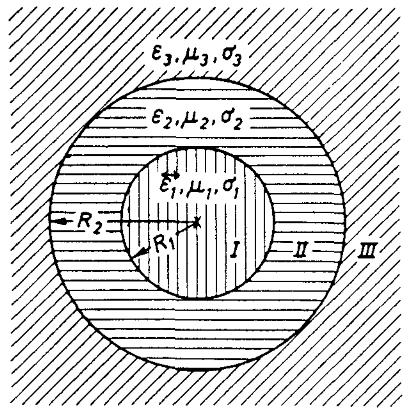

Fig.1: The three-layer spherical resonator.

The permittivities, permeabilities and conductivities of all layers may be of arbitrary value. Using this model of a spherical resonator the following microwave resonators can be discussed by one theory (Fig.2):

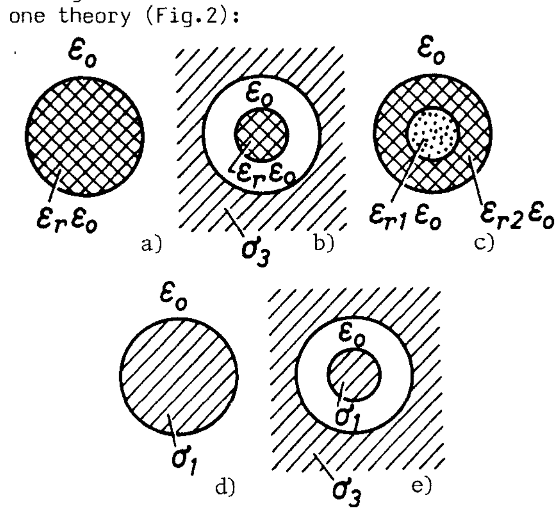

Fig.2: Several spherical resonator structures, which can be analyzed, using the described field theory.

- The free isotropic and anisotropic dielectric sphere in vacuum (Fig.2a)
- The spherical cavity with or without an inner dielectric sphere (Fig.2b)
- The spherical cavity with or without an inner dielectric sphere (Fig.2c)
- The free conducting sphere of one or two layers (Fig.2d)
- The spherical cavity with or without an inner conducting sphere (Fig.2e) etc..

RESULTS

Several problems can be solved using this model of a spherical resonator, which could not be discussed before, e.g. the two layer dielectric cavity or open resonator can be used to analyze and optimize the dimensions and the Q- factor of this stucture by using two materials of different dielectric constants. If the structure in Fig.2b is considered, for the first time the correct convergence of the cavity modes into the modes of the free dielectric sphere can be studied by changing the conductivity of layer 3 from very high values (metal) to zero (vacuum).

Only some typical results can be discussed here. If the isotropic resonators are considered, the H_{110}-mode is the fundamental mode in the case of the free dielectric sphere and the E_{110}-mode is the fundamental mode in the case of the cavity resonator.

In Fig.3a) typical results for the resonant frequency of a two-layer dielectric resonator in vacuum is shown for a constant dielectric constant of the inner sphere in dependence on the ratio of the radius r_1 of the inner sphere to the radius r_2 of the outer sphere with r_2=3.5mm=const..

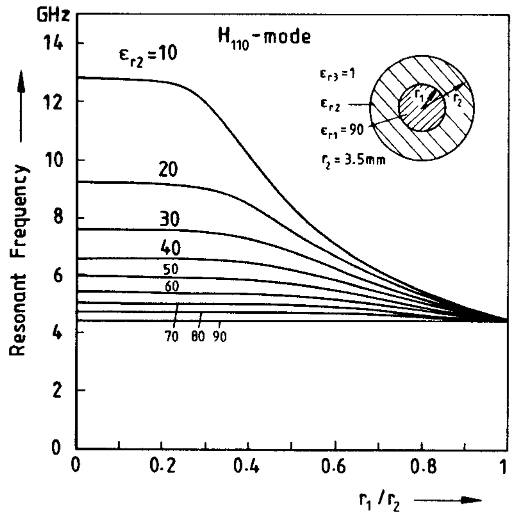

Fig.3a): Dependence of the resonant frequencies of a two layer dielectric resonator in vacuum on the ratio of the sphere radii and with the dielectric constant as a parameter.

Fig. 3b) shows the adjoint Q-factors of the resonators for the case of lossless dielectric materials, i.e. that the losses are pure radiation losses. As can be seen from Fig.3b) there is no optimal radius-ratio for which the Q-factor e.g. is maximum; the highest Q-factor is found for the case of the resonator with only one layer which additionally has the highest value of the dielectric constant. The Q-factor of the two-layer dielectric resonator continuously increases with an increasing (medium) dielectric constant of the resonator, i.e. with increasing radius-ratio. This result may change in the case, that the dielectric losses are considered additionally and the loss factors of the two layers differ by a large amount.

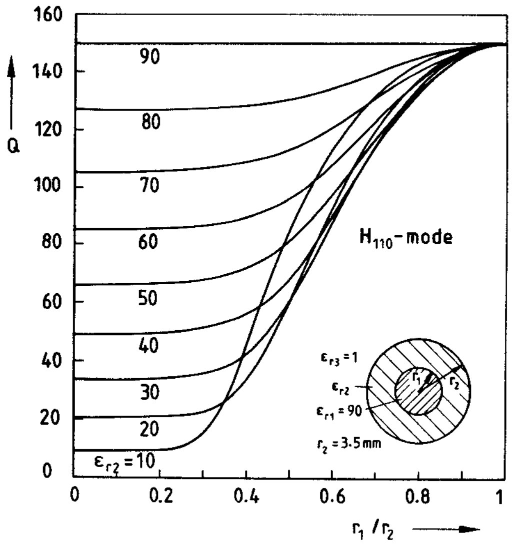

Fig.3b): The Q-factors of a two-layer dielectric resonator in vacuum in dependence on the radius-ratio and with the dielectric constant as parameter.

The results for the resonant frequencies of a closed cavity behave in a similar way as in the case of the free resonator (Fig.4a); as mentioned above, the fundamental mode of this resonator is the E_{110}-mode.

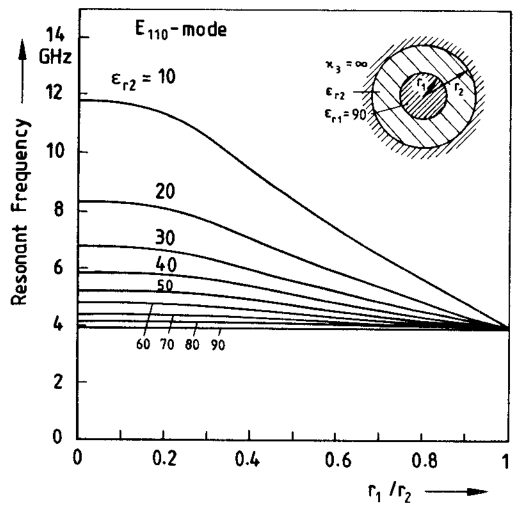

Fig.4a): Dependence of the resonant frequencies of a cavity resonator with two dielec-layers on the radius-ratio and with the dielectric constant as a parameter.

The Q-factors (again for lossless dieletric materials) on the other side show a clear maximum in dependence on the radius r_1 of the inner sphere,

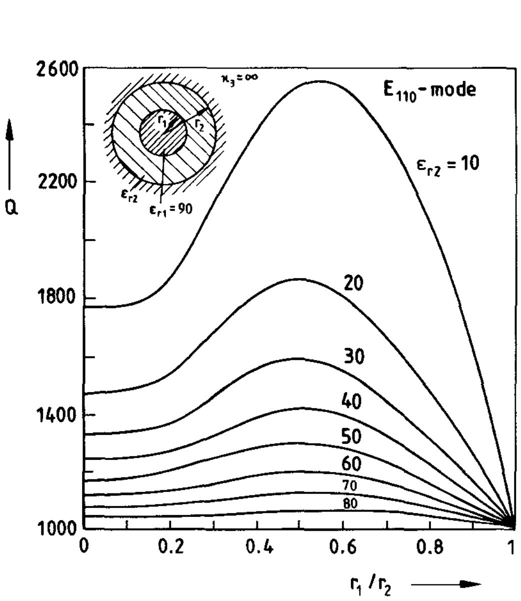

Fig.4b): The Q-factors of a two-layer dielectric cavity in dependence on the radius-ratio and with the dielectric constant as a parameter.

if the radius r_2 of the outer sphere is kept constant. This optimum value is determined by the field concentration of the electric field inside the inner sphere and the minimum current density in the conducting walls of the cavity. This optimum value is essential for many applications e.g. in measurement techniques, where high Q-factors are needed.

If the inner sphere e.g. of the cavity shown in Fig. 2b is uniaxially anisotropic, as described in the first part of this paper, the threefold degeneracy of the resonant frequency is broken up and at least two different resonant frequencies can be measured for the E_{mn0}-modes dependent on the orientation of the main crystal axis with respect to the excited electromagnetic field inside the spherical resonator. Fig.5 shows the measured resonant frequencies of an uniaxially anisotropic sphere in a spherical cavity for different orientations of the spherical insert with respect to the exciting field probes.

In equ.(9) it is assumed, that the main crystal axis is parallel to the z-axis of the coordinate system. As the field equations show, The E_{mn0}-modes then only have a H_φ-, an E_r-, and an E_ϑ-component, but by the additional coupling between these three field components, as it is e.g. described by equ.(3), compared to the isotropic resonator, the field distribution is much more complicate. It has already been shortly described in equ.(12) how a solution of the complicate field problem can be found. As equ.(9) clearly shows, the resonant frequencies of the modes which are excited with an electric field parallel to the z-axis are influenced by both parameters of the permittivity tensor.

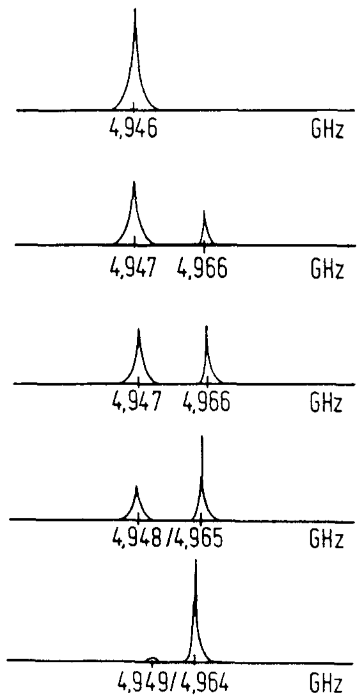

Fig.5: Measured resonance frequencies of an anisotropic spherical cavity ($Ca_3(VO_4)_2$) for different orientations of the spherical dielectric insert inside the cavity.

Applications of spherical resonators can be found in all microwave structures, where dielectric resonators are used; especially in the millimeter-wave region spheres are advantageous because they easily can be produced precisely and with optical surfaces. The isotropic and anisotropic spherical three-layer resonator can be used for the accurate measurement of the conductivity and permittivity of isotropic and anisotropic dielectric materials.

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